CS130A

Review

12/7/2010

Outline

- Performance analysis
- LZW compression
- Heap
- RB-tree
- B-tree
- Union-Find
- Sorting

Big-Oh

- f(N) = O(g(N))
- There are positive constants c and n_0 such that

  $f(N) <= c\ g(N)$ when $N >= n_0$
- The growth rate of f(N) is less than or equal to the growth rate of g(N)
- g(N) is an upper bound on f(N)
- Example
  - Let $f(N) = 2N^2$. Then
    - $f(N) = O(N^4)$
    - $f(N) = O(N^3)$
    - $f(N) = O(N^2)$ (best answer, asymptotically tight)

General Rules (algorithm analysis)

- For loops
  - at most the running time of the statements inside the for-loop (including tests) times the number of iterations.
- Nested for loops

```
for (i=0;i<n;i++)
      for (j=0;j<n;j++)
            k++;
```

  - the running time of the statement multiplied by the product of the sizes of all the for-loops
  - $O(N^2)$
- Consecutive for loops

```
for (i=0;i<n;i++)
      a[i]=0;
for (i=0;i<n;i++)
      for (j=0;j<n;j++)
            a[i] += a[j]+i+j;
```

  - These just add
  - $O(N) + O(N^2) = O(N^2)$

Exercise 2.1[w]

- Order the following functions by growth rate:
  - N, $N^{1.5}$, N^2, $N\log N$, $N\log\log N$, $N\log^2 N$, $N\log(N^2)$, $2/N$, 2^N, 37, $N^2\log N$, N^3

- Answer:

$2/N$, 37, N, $N\log\log N$, $N\log N$, $N\log N^2$, $N\log^2 N$, $N^{1.5}$, N^2, $N^2\log N$, N^3, 2^N

Exercise 2.7[w]

- Give an analysis of the Big-Oh running time for the following program fragments:

a

```
sum = 0;
for( i=0; i<n; i++ )
    for( j=0; j<n*n; j++ )
        sum++;
```

b

```
sum = 0;
for( i=0; i<n; i++ )
    for( j=0; j<i; j++ )
        sum++;
```

c

```
sum = 0;
for( i=0; i<n; i++ )
    for( j=0; j<i*i; j++ )
        for( k=0; k<j; k++)
            sum++;
```

d

```
sum = 0;
for( i=1; i<n; i++ )
    for( j=1; j<i*i; j++ )
    if( j%i == 0 )
        for( k=0; k<j; k++ )
            sum++;
```

Exercise 2.27[w]

- The input is an N by N matrix of numbers that is already in memory. Each individual row is increasing from left to right. Each individual column is increasing from top to bottom. How would you decides if a number X is in the matrix? What is its worst-case complexity in Big-Oh

LZW algorithm

- Encoding:
  - Longest Prefix Matching(eg. matched string X) in the code table
    - Output the code for X
  - Add a new entry for string Xc, c is a char
  - Move pointer to c's position
  - Repeat
- Example:
  - abbbaaab

LZW algorithm

Output: string(first CodeWord);
 while(there are more CodeWords){
    if(CurCodeWord is in the Dict)
      output: **string(CurCodeWord)**
    else
      output: **PreviousOutput + FC(PreviousOutput);**
   insert in the Dictionary: **PreviousOutput + FC(CurrentOutput);**
}

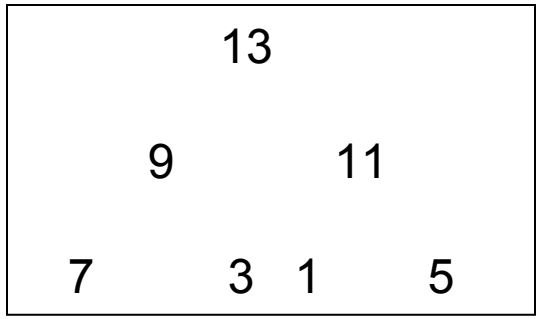
Example:
   0 2 3 2 4

Heap

- Usually in array representation
- Key (for Max-Heap):
  - For any node v and its parent p, p.key>=v.key
- Insert
  - Put at the end of the array. Bottom up
  - Delete/Extract Max: Replace the root with the last element. Top-down fix

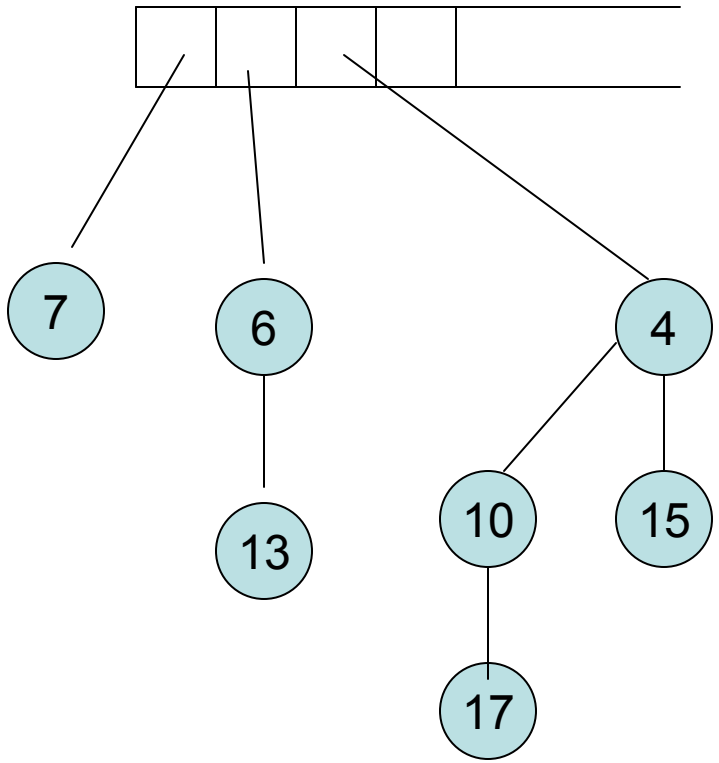
Heap

- Example
  - We have the following elements in array X
    - X = [1 3 5 7 9 11 13]
  - Transform the array into a max heap in O(n) time. Draw the resulting max heap



Binomial Heap

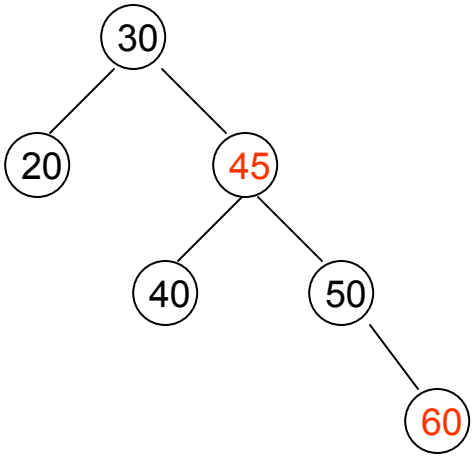
- Delete the smallest element from the following Binomial Heap



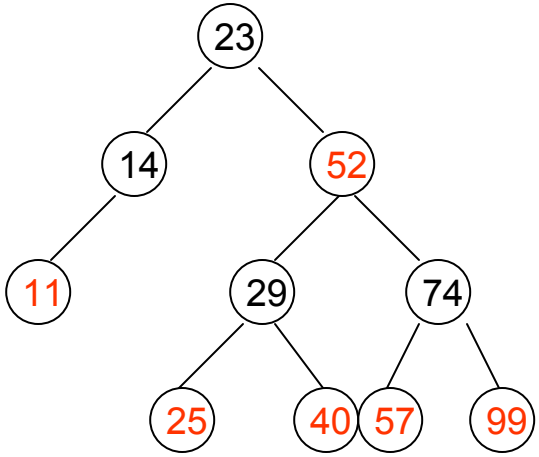
Binary Heap & Binomial Heap

Procedure	Binomial Heap	Binary Heap
Make-Heap	O (1)	O (1)
Insert	O (log n)	O (log n)
Minimum	O (log n)	O (1)
Extract-Min	O (log n)	O (log n)
Merge	O (log n)	O (nlog n) -> O (n)
Decrease-Key	O (log n)	O (log n)
Delete	O (log n)	O (log n)

Exercise

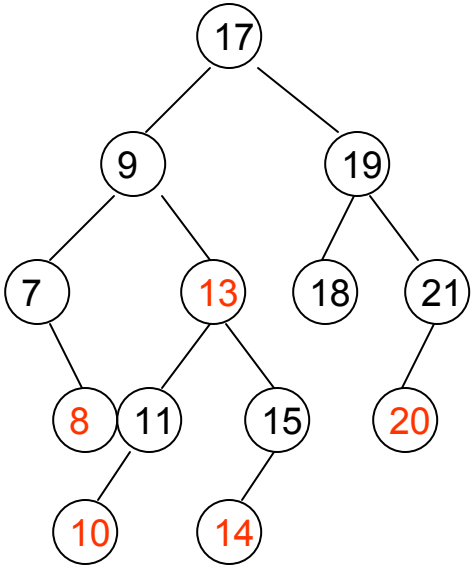


Insert 70, 80



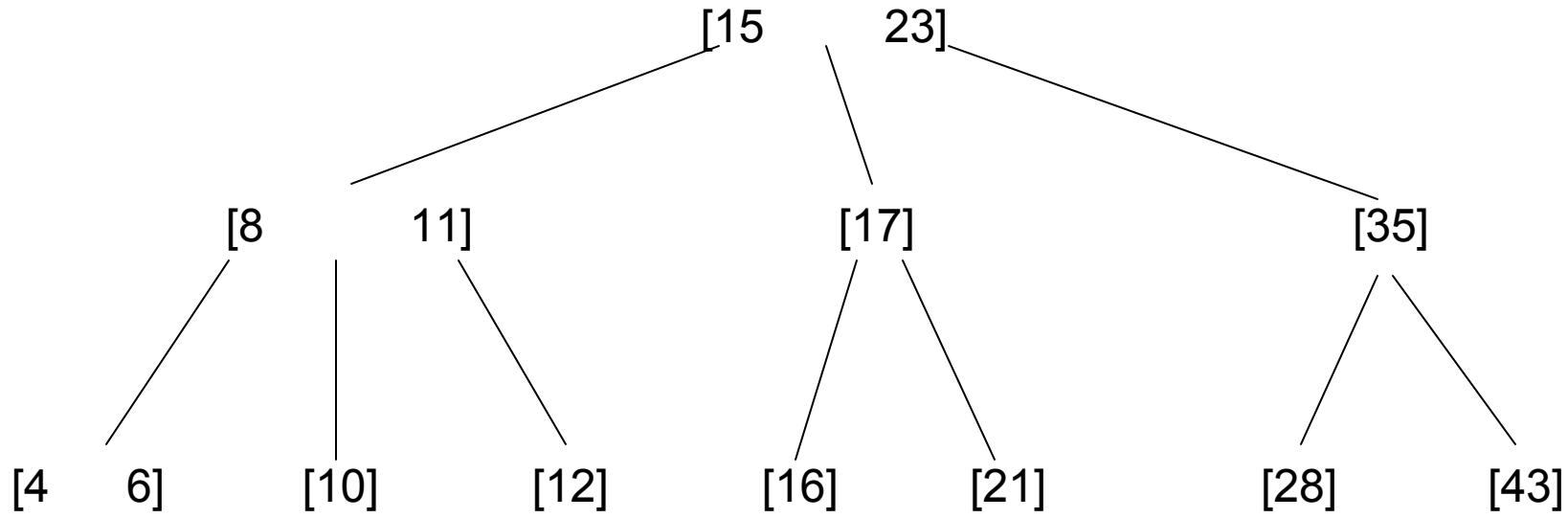
Insert 26

Exercise



Delete 18

B-tree delete exercise



Delete 16

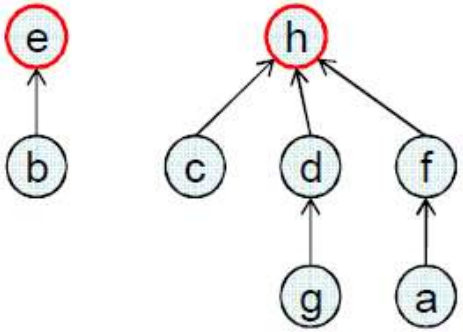
- *Borrow from right sibling*
- *Exchange with smallest of right sub-tree*

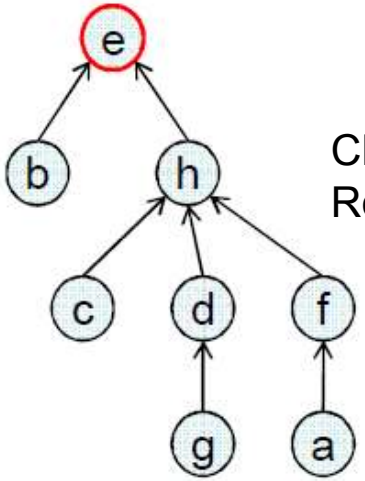
Union-find

- Used to maintain equivalent classes.
- Union(x,y): Group x and y into one class
  - Initially each element is a class. N different classes in total
- Find(x): Returns the representative of x's class
  - If x and y belongs to same class, Find(x) == Find(y)
- Binary Tree/Array Representation.
- Two optimizations:
  - For Union: Weighted Union
  - For Find: Path compression

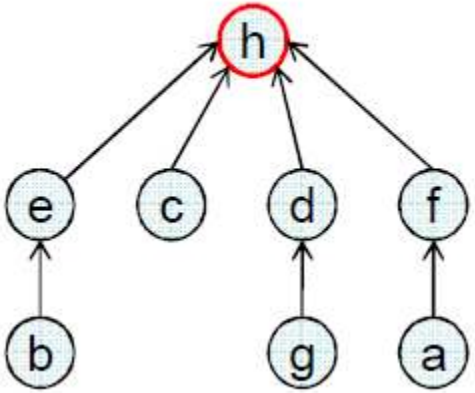
Union by rank

- Tow choices for the union operation





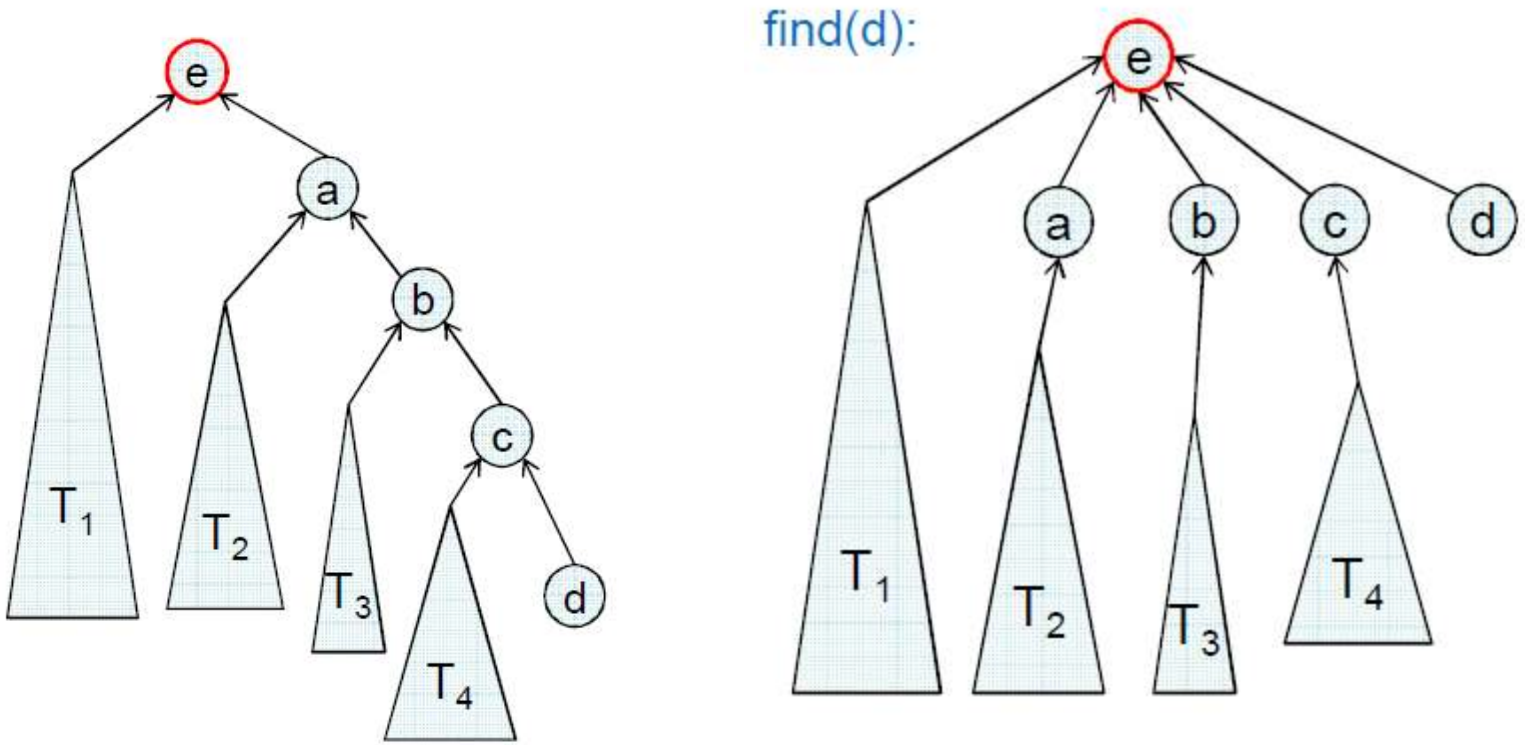
Choice 1:
Root = e



Choice 2:
Root = h

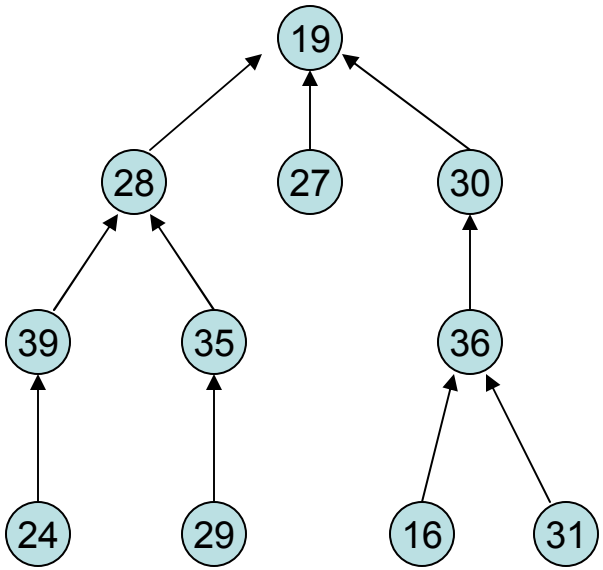
Path compression

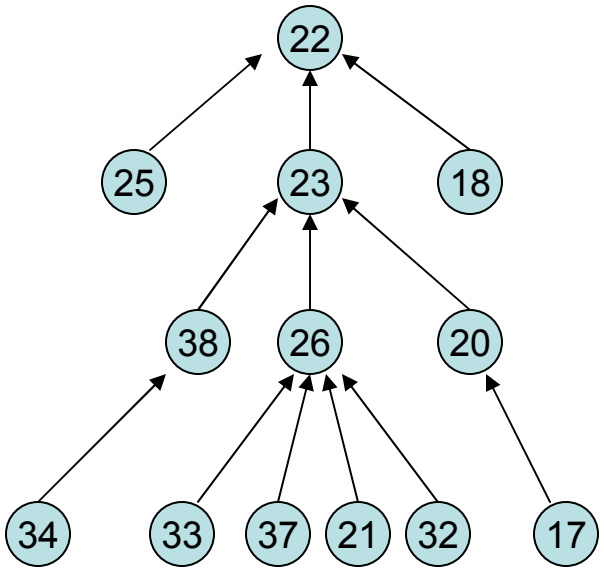
- A little extra computation performed during find, that makes the tree shorter



Exercise

- Show the resulting tress after each union-find operations. Use weighted union & path compression





Find(21), Find(18), Find(36), Find(26), Find(23), Union(19,22)

Heap Sort

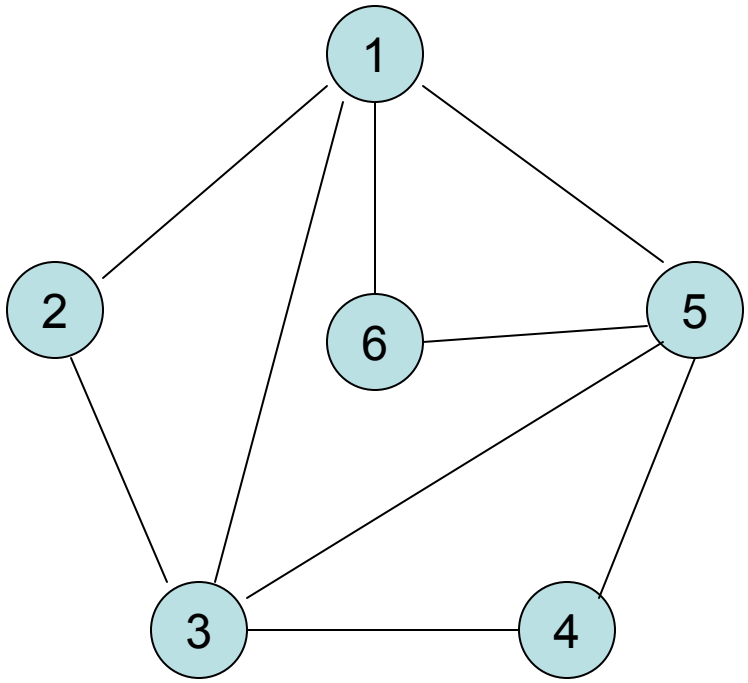
- Sort the following array
  - 19, 31, 23, 66, 73, 79, 45, 87
  - Show the array after each iteration

Euler Circuit

- Euler circuit
  - Start from a vertex v, travel all edges exactly once and come back to v
  - Euler theorem: If the degree of any vertex is even, the graph exists an EC
  - Barnard's algorithm
  - Euler(v)
    - For vertex w that is adjacent to v and (v,w) is not marked
      - Mark (v,w)
      - Path = (v,w) + Euler(w) + Path
    - Return Path

Euler Circuit

- Example



Reference

- Shuo Wu's review
  - http://www.cs.ucsb.edu/~teo/cs130a.f10/review.pdf
- ucsd cs101
  - http://cseweb.ucsd.edu/~dasgupta/101/lec9.pdf
- sjsu cs146
  - *www.cs.sjsu.edu/~lee/cs146/24CS146JC**Merge**.ppt*